Shock waves in a dusty plasma

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In the present paper, shocks in a dusty plasma are studied. A set of macroscopic shock equations are introduced in which the charge on the dust particles and the electrostatic potential in the dusty plasma have been taken into account. It is found that two shock modes may exist in a dusty plasma. One that is called the fast mode corresponds to an acoustic wave and the other, called the slow mode, corresponds to a dust-acoustic wave. The results show that the electrostatic energy of dust particles affects the shock speed and shock heating. The shock speed of the slow mode is found to be strongly dependent on the nondimensional dust density parameter p. In a tenuous dust plasma of $p \ll 1$ for the slow mode, the flow of energy as the shock heats is mainly converted into thermal energy of the dust particles; for the fast mode, the plasma heating may be reduced and partly transferred into an increase of the charge on the dust particles when they cross the shock.

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I. INTRODUCTION

Dusty plasma, a medium consisting of micron and submicron particles, exist in space, in the earth's atmosphere, in planetary rings, in the interstellar medium, etc., and also in man-made environments such as some production processes, rocket exhausts, and in many laboratory experiments. The dust particles in a plasma become electrically charged and interact with each other, as well as with the ions and electrons of the plasma. The charge on the particles depends on their radius and chemical composition, and also on their physical environment, and it varies with changes in its environment. A study of dusty plasma has led to the discovery of different wave modes, instabilities, and crystallization (''plasma crystals'') etc. [1]. The supersonic flow in a dusty plasma and shocked dusty plasma by some wave modes have also been studied [2,3]. The Mach cones, or *V*-shaped disturbances created by supersonic objects in dusty plasma were predicted [4,5] and recently detected in laboratory dusty plasma in a state of crystalline Coulomb lattice [6]. In the present paper, the general character of shocks in a dusty plasma is investigated. We include the charge on the dust particles and the electrostatic potential of a dusty plasma. Indeed, the electrostatic energy of the dust particles may be a major part of total energy and play an important role for the dynamics of the dust particles that may have an effect on the shock by its energy conversion.

We consider plane shocks moving in a dusty plasma in a direction normal to the plane of the shock (Fig. 1). The fluid undergoes a discontinuity in the fluid and also the electrostatic parameters. The macroscopic conservation equations integrated across the shock give a set of equations relating the fluid properties on either side, which is independent of the interior structure of the shock. In Sec. II this set of macroscopic equations is introduced and used to study the shock modes. The shock speed and shock heating are studied in Secs. III and IV, respectively, and the results are discussed in Sec. V.

II. SHOCK MODES IN A DUSTY PLASMA

We study a dust plasma that contains three components: negatively charged dust grains, electrons, and positive ions. The ions and electrons are assumed to be closely coupled to each other due to frequent collisions. On the other hand, the dust grains may be discoupled from the plasma due to weak collision effects because of their large mass and, in some cases, also because of their low-number density compared with ions and electrons. We therefore use a model in which the plasma fluid and the dust grain fluid are dealt with separately.

The dust particles move together with the plasma stream at the same velocity in the unshocked region. They enter the front of the shock with a velocity U_1 (Fig. 1). From the conservation of mass and momentum equations, one finds the jump conditions for the ion-electron fluid across the shock to be

$$J\{V\}-\{U\}=0, \tag{1}$$

$$\{P\}+J\{U\}+\overline{\rho}\{\Phi\}+\overline{\Phi}\{\rho\}=0, \tag{2}$$

and for the dust fluid

$$J_d\{V_d\}-\{U_d\}=0, \tag{3}$$

$$\{P_d\}+J_d\{U_d\}+\overline{\rho_d}\{\Phi\}+\overline{\Phi}\{\rho_d\}=0. \tag{4}$$

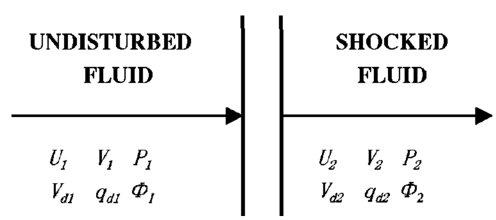

FIG. 1. A plane shock moving in a dusty plasma where the fluid and electrostatic parameters undergo a discontinuity.

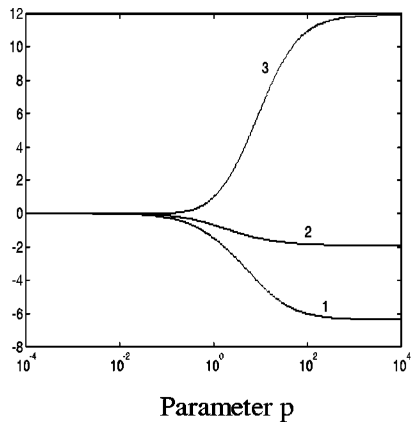

FIG. 2. Electrostatic potential and dust charge density of a dust cloud in hydrogen plasma varies as a function of the parameter p. Curve one is for $\rho_d/n_0 e$, curve two for $e\Phi/k_B T$, and curve three for $\rho_d \Phi/n_0 k_B T$.

Here, $\{A\}=(A_2-A_1)$ is the usual notation for a shock, the subscripts 1 and 2 stand for the undisturbed and shocked fluid, respectively, a subscript d stands for the dust grain component. V, V_d the specific volume, is the reciprocal of the mass density $V=1/nm_i$, $V_d=1/n_d m_d$; $J=U/V$, $J_d=U_d/V_d$ is the flux density; P, P_d the thermal pressure; Φ the electrostatic potential; and ρ, ρ_d the charge density, and a bar is for an average

$$\overline{\rho}=\frac{\rho_2+\rho_1}{2}, \quad \overline{\Phi}=\frac{\Phi_2+\Phi_1}{2}.$$

Equations (1)–(4) may be looked upon as a set of linear homogeneous equation of the variables $\{V\}$, $\{U\}$, $\{U_d\}$, and $\{\Phi\}$. A condition for the nonzero solution of Eqs. (1)–(4) is

$$\begin{vmatrix} J & -1 & 0 & 0 \\ \frac{\{P\}}{\{V\}} & J & 0 & \overline{\rho}+\overline{\Phi}\frac{\{\rho\}}{\{\Phi\}} \\ \frac{\{P_d\}}{\{V\}} & 0 & J_d & \overline{\rho_d}+\overline{\Phi}\frac{\{\rho_d\}}{\{\Phi\}} \\ 0 & 0 & -1 & J_d\frac{\{V_d\}}{\{\Phi\}} \end{vmatrix}=0. \tag{5}$$

Using the condition of charge neutralization in the fluid, i.e., $\rho=-\rho_d$, we find from Eq. (5)

$$\alpha^2\frac{\{V_d\}}{\{\Phi\}}J^4+\left(\overline{\rho_d}+\overline{\Phi}\frac{\{\rho_d\}}{\{\Phi\}}+\alpha^2\frac{\{V_d\}}{\{\Phi\}}\frac{\{P\}}{\{V\}}\right)J^2+\left(\overline{\rho_d}+\overline{\Phi}\frac{\{\rho_d\}}{\{\Phi\}}\right)\frac{\{P+P_d\}}{\{V\}}=0, \tag{6}$$

where

$$\alpha=\frac{J_d}{J},$$

is the ratio between the flux density of the dust particles stream and the plasma stream.

The solution of Eq. (6) is

$$J_{\pm}^2=\frac{-\left(\overline{\rho_d}+\overline{\Phi}\frac{\{\rho_d\}}{\{\Phi\}}+\alpha^2\frac{\{V_d\}}{\{\Phi\}}\frac{\{P\}}{\{V\}}\right)}{2\alpha^2\frac{\{V_d\}}{\{\Phi\}}}\pm\frac{\sqrt{\left[\left(\overline{\rho_d}+\overline{\Phi}\frac{\{\rho_d\}}{\{\Phi\}}+\alpha^2\frac{\{V_d\}}{\{\Phi\}}\frac{\{P\}}{\{V\}}\right)^2-4\alpha^2\left(\overline{\rho_d}+\overline{\Phi}\frac{\{\rho_d\}}{\{\Phi\}}\right)\frac{\{V_d\}}{\{\Phi\}}\frac{\{P+P_d\}}{\{V\}}\right]}}{2\alpha^2\frac{\{V_d\}}{\{\Phi\}}}. \tag{7}$$

In a dusty plasma, it is convenient to introduce a dimensionless parameter p defined by [7],

$$p=\frac{n_d a_d k_B T}{n_0 e^2},$$

which is related to the ratio of dust space charge to the electron space charge, where a_d is the radius of the dust particles, k_B is the Boltzmann constant, e is the electronic charge, and n_0 refers to the common plasma density value when $\Phi=0$. A dusty plasma is tenuous in dust when $p\ll 1$ and dense when $p\gg 1$. The local electrostatic potential Φ will, in the following, be calculated by Eq. (14). The dust particle charge $q_d=a_d\Psi$ is related to Ψ, which is the dust surface potential relative to the Φ plasma potential. Ψ may be calculated by Eq. (13). In Fig. 2, the curves 1, 2, and 3 show $\rho_d/n_0 e[=pF_\Psi(p)]$, $e\Phi/k_B T$, and $\rho_d\Phi/n_0 k_B T$ as a function of the parameter p for a dusty hydrogen plasma.

For the shocks $\{n_d\}>0$, $\{V\}<0$, $\{V_d\}<0$, $\{P\}>0$, $\{P_d\}>0$, and $\{T\}>0$, one finds that $\{\Phi\}<0$, $\{p\}>0$,

$$\frac{\{P\}}{\{V\}}<0, \quad \frac{\{P_d\}}{\{V\}}<0,$$

$$\frac{\{V_d\}}{\{\Phi\}}>0,$$

and

$$\left(\overline{\rho}_d+\overline{\Phi}\frac{\{\rho_d\}}{\{\Phi\}}\right)=\frac{\{\rho_d\Phi\}}{\{\Phi\}}<0.$$

We can see that there are the two positive solutions of Eq. (7). It means that there are two shock modes in a dusty plasma. They are called the *fast mode* for $J=J_+$ and *slow mode* for $J=J_-$, and corresponds to the acoustic wave and the dust-acoustic wave, respectively, as will be seen in the next section.

III. SHOCK SPEED IN A DUSTY PLASMA

When $P\gg P_d$, we find from Eq. (7)

$$J_+^2=-\frac{\{P\}}{\{V\}}+\frac{1}{\alpha^2}\frac{\{\rho_d\Phi\}}{\{V_d\}}\frac{\{P_d\}}{\{P\}},\tag{8}$$

and

$$J_-^2=-\frac{1}{\alpha^2}\frac{\{\rho_d\Phi\}}{\{V_d\}}\left(1+\frac{\{P_d\}}{\{P\}}\xi\right),\tag{9}$$

here,

$$\xi=\begin{cases}1 & \text{if}\quad \left|\frac{\{P\}}{\{V\}}\right|\geq\left|\frac{1}{\alpha^2}\frac{\{\rho_d\Phi\}}{\{V_d\}}\right|,\\ -\frac{\{P\}}{\{V\}}\left[\frac{1}{\alpha^2}\frac{\{\rho_d\Phi\}}{\{V_d\}}\right]^{-1} & \text{if}\quad \left|\frac{\{P\}}{\{V\}}\right|\leq\left|\frac{1}{\alpha^2}\frac{\{\rho_d\Phi\}}{\{V_d\}}\right|.\end{cases}$$

A weak shock may be treated as a perturbation of an isotropic process. Assuming that the plasma and the dust particles are perfect gases, one has

$$\frac{\{P\}}{\{V\}}=-\frac{C_s^2}{V^2},$$

and

$$\frac{\{P_d\}}{\{V_d\}}=-\frac{C_d^2}{V_d^2},$$

where $C_s=(\gamma PV)^{1/2}$ is the speed of sound and $C_d=(\gamma_d P_d V_d)^{1/2}$, where γ is the ratio of the special heats. Then, it may be found from Eqs. (8) and (9) that

$$J_+^2=\frac{C_s^2}{V^2}\left(1-\frac{C_d^2}{C_s^2}\frac{\{\rho_d\Phi\}}{\{P\}}\right)\tag{10}$$

and

$$J_-^2=\frac{C_d^2}{V^2}\left(\frac{\{\rho_d\Phi\}}{\{P_d\}}+\frac{\{\rho_d\Phi\}}{\{P\}}\xi\right).\tag{11}$$

First, we discuss the fast mode of Eq. (10). According to $C_s^2/C_d^2=m_i/m_d$ and Eqs. (13), (14), and (16) for a tenuous dust cloud with $p\ll1$, we have that $\rho_d\Phi\sim pP$ and we find $(C_d^2/C_s^2)(\{\rho_d\Phi\}/\{P\})\sim(m_i/m_d)p\ll1$. For a dense dust cloud with $p\gg1$ and $\rho_d\Phi\sim P$, we find $(C_d^2/C_s^2)(\{\rho_d\Phi\}/\{P\})\sim(m_i/m_d)\ll1$. The second term in the right-hand side of Eq. (10) may be neglected and we have the shock speed

$$U_+^2=J_+^2V^2\cong C_s^2.\tag{12}$$

This means that the shock speed of the fast mode is the speed of sound. It is the same as that in a normal plasma flow. The dust particles have not affected the shock speed of the fast mode.

For the slow mode, due to $P\gg P_d$ and $|\xi|\leq1$, the first term in the bracket in the right-hand side of expression Eq. (11) is much larger than the second term that may be neglected. The density of the electric energy $\rho_d\Phi$ may be calculated by using the following rational function approximations for the dust and plasma potentials [7]

$$\frac{e\Psi}{k_BT}=\frac{a_0+a_1p}{1+b_1p+b_2p^2}\equiv F_\Psi(p),\tag{13}$$

$$\frac{e\Phi}{k_BT}=\frac{c_1p+c_2p^2}{1+d_1p+d_2p^2}\equiv F_\Phi(p).\tag{14}$$

We further have that

$$q_d=a_d\Psi=\frac{a_dk_BT}{e}F_\Psi(p)\tag{15}$$

and

$$\rho_d\Phi=pF_\Psi(p)F_\Phi(p)n_0k_BT.\tag{16}$$

For a weak shock, one has

$$\frac{\{\rho_d\Phi\}}{\{P_d\}}=\frac{d(\rho_d\Phi)}{dP_d}.$$

Using Eq. (16) and treating the shock as an isotropic process from Eq. (11) we find the shock speed of the slow mode to be

$$U_-^2=J_-^2V^2=C_d^{*2}\left\{\frac{1}{pF_\Psi^2(p)}\frac{d[pF_\Phi(p)F_\Psi(p)]}{dp}+\frac{F_\Phi(p)}{pF_\Psi(p)}\left(1-\frac{1}{\gamma_d}\right)\right\}.\tag{17}$$

Here, C_d^* is the velocity of the dust-acoustic wave [8]

$$C_d^{*2}=\omega_d^2\lambda_D^2,$$

where, $\omega_d^2=4\pi n_dq_d^2/m_d$, $1/\lambda_D^2=1/\lambda_{De}^2+1/\lambda_{Di}^2\approx4\pi n_0e^2/k_BT$.

Figure 3 shows the speed of slow mode shock, in units of the dust-acoustic wave velocity, as a function of the parameter p of Eq. (17). Three case are calculated: the solid line for a hydrogen ion (proton) plasma, the dashed line for a singly

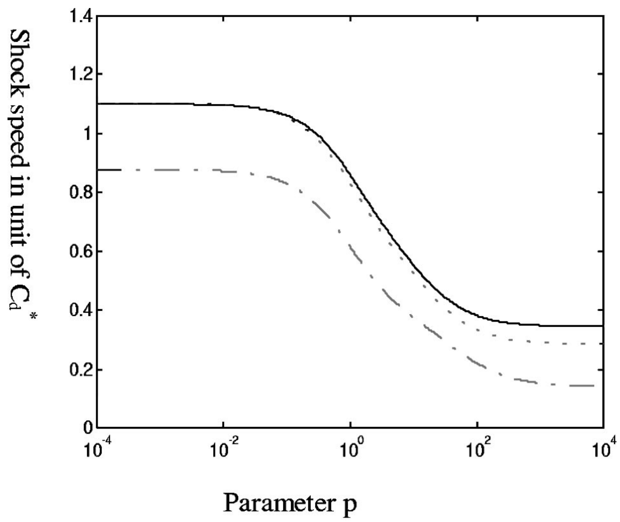

FIG. 3. The speed of the slow mode shock in a dust plasma (in terms of the velocity of dust-acoustic wave C_d^*), as a function of the parameter p, based on Eq. (17). The solid line is for hydrogen plasma, the dashed line for single-ionized oxygen (O^+) plasma, and the dashed-dot line for double-ionized sulphur (S^{++}) plasma.

ionized oxygen (O^+) plasma, and the dashed-dot line for a doubly ionized sulphur (S^{++}) plasma, respectively. The coefficients in Eqs. (13) and (14) for the three cases are shown in Table I and $\gamma_d=5/3$ is assumed.

It can be seen from Fig. 3 that the speed of the slow mode shock depends on the parameter p.

IV. SHOCK HEATING IN A DUSTY PLASMA

A jump condition of the shock from the energy equation is

$$\left\{\left(I+\frac{1}{2}U^2\right)J+\left(I_d+\frac{1}{2}U_d^2\right)J_d+(P+\rho\Phi)U+(P_d+\rho_d\Phi)U_d\right\}=0. \tag{18}$$

The internal energy density per mass unit of the plasma is given by

$$I=\frac{PV}{(\gamma-1)}, \tag{19}$$

and the internal energy density per mass unit of the charged dust particle is

$$I_d=\frac{1}{M_d}\frac{q_d^2}{4\pi a_d}+\frac{P_dV_d}{(\gamma_d-1)}. \tag{20}$$

For simplicity, we use a tenuous dust plasma of $p\ll 1$ as an example to show how the electrostatic energy of the dust particles affect the shock heating. We will, in the following, discuss details of the two different models separately.

A. Slow mode shock

In this case, the shock speed is in a region of $C_s \gg U_1 > U_-$. The internal energy of the plasma should not be affected by the shocked fluid due to the shock speed being much less than the thermal velocity of the plasma particles. Therefore,

$$\{T\}\cong 0, \tag{21}$$

$$\{I\}\cong 0, \tag{22}$$

and from Eq. (15)

$$\{q_d^2\}=\left(\frac{a_dk_BT}{e}\right)^2\{a_0^2\}\cong 0. \tag{23}$$

We have used that for a tenuous dust plasma of $p\ll 1$ then $F_\Psi(p)=a_0$. From $F_\Phi(p)=c_1p$, for the tenuous dust plasma, one has

$$\frac{\rho_d\Phi U_d}{U_d^2J_d}=\frac{a_d(k_BT)}{e^2}a_0c_1p\ll 1, \tag{24}$$

and

$$\frac{\rho\Phi U}{U^2J}=\frac{n_dm_d}{nm_i}\frac{C_d^2}{U^2}a_0c_1p\ll 1. \tag{25}$$

We then find from Eqs. (18)–(25)

$$\left\{\left(\frac{\gamma_dC_d^2}{\gamma_d-1}+\frac{U_d^2}{2}\right)J_d+\frac{U^2}{2}J\right\}=0. \tag{26}$$

We may now show that for a slow mode, the flow energy is converted by the shock into thermal energy of the dust particles. We have from Eq. (26) that

$$\frac{T_{d2}}{T_{d1}}=\left[1+\frac{(\gamma_d-1)}{2\gamma_d}\left(1+\alpha-\frac{1}{r^2}-\alpha\frac{1}{r_d^2}\right)\right]\mathcal{M}_d^2, \tag{27}$$

where $r=U_1/U_2=D_2/D_1$, and $r_d=U_{d1}/U_{d2}=D_{d2}/D_{d1}$ is the compression ratio of the plasma and the dust, respec-

TABLE I. The coefficients in Eqs. (13) and (14) for three cases of dusty plasma (Z_i is the ion charge number and A_i is the atomic weight).

Z_i	A_i	a_0	a_1	b_1	b_2	c_1	c_2	d_1	d_2
1.0	1.0	−2.50	−0.764	1.09	0.120	−1.26	−0.210	1.04	0.112
1.0	16.0	−3.61	−1.07	1.11	0.0838	−1.83	−0.246	1.14	0.0956
2.0	32.0	−3.43	−0.509	0.686	0.0138	−1.10	−0.0666	1.13	0.0364

tively, $\mathcal{M}_d = U_1/C_{d1}$ is the dust Mach number in region one. Since r and $r_d > 1$, $\gamma_d > 1$, the large square in the parentheses in Eq. (27) will be larger than one. Since also the Mach number is larger than one, we find that the dust gas may be greatly heated when it crosses a slow shock.

B. The fast mode shock

In this case, $U_1 \gg C_{s1}$, and the plasma will be heated by the shock. For the tenuous dust plasma, we have from Eq. (23) that

$$\{q_d^2\} = \left(\frac{a_d a_0}{e}\right)^2 \{(k_B T)^2\}. \tag{28}$$

Equations (18)–(20) combined with Eqs. (24), (25), and (28) gives

$$\frac{\gamma C_{s1}^2}{\gamma-1} + (1+\alpha)\frac{U_1^2}{2} + \frac{\alpha}{M_d}\left(\frac{q_{d1}^2}{4\pi a_d} + W_{d1}\right) = \frac{\gamma C_{s2}^2}{\gamma-1} + \frac{U_2^2}{2} + \frac{\alpha}{M_d}\left(\frac{q_{d2}^2}{4\pi a_d} + W_{d2}\right) + \alpha\frac{U_{d2}}{2}. \tag{29}$$

Dividing Eq. (31) by C_{S1}^2 and for a tenuous dust plasma of $p \ll 1$ neglecting $W_d(=P_d V_d/\gamma_d - 1)$, the thermal energy of the dust particles, we find

$$\frac{T_2}{T_1} = \frac{1 + \dfrac{(\gamma-1)}{2\gamma}\left(1 + \alpha - \dfrac{1}{r^2} - \alpha\dfrac{1}{r_d^2}\right)}{(1+\alpha\delta)}\mathcal{M}^2, \tag{30}$$

here, $\mathcal{M} = U_1/C_{s1}$ is the plasma Mach number in region one. If $\alpha = 0$, that is, no dust in the plasma, Eq. (30) will describe the case of a ordinary plasma heated by a shock. The factor δ in the denominator of Eq. (30) is

$$\delta = \frac{|a_0|}{4\pi}\frac{m_i}{M_d}\{Z_d\}, \tag{31}$$

where $\{Z_d\} = |q_{d2}|/e - |q_{d1}|/e$ is the change of the charge number on a dust particle when it crosses the shock. For the tenuous dust plasma

$$\{Z_d\} = |a_0|\frac{a_d k_B (T_2 - T_1)}{e^2}, \tag{32}$$

and it can be seen that $\{Z_d\} \gg 1$ due to the plasma heating by the shock. Equation (30) shows that the plasma heating is reduced due to the charge increase of the dust particles when they cross the shock.

V. CONCLUSION AND DISCUSSION

From the above, it can be seen that a shock in a dusty plasma differs from that in an ordinary plasma in the following ways.

(1) There are two shock modes in a dusty plasma corresponding to two wave modes. The fast mode corresponds to the acoustic wave, which also exists in the ordinary plasma. The other, slow mode, corresponds to the dust-acoustic wave, which is unique for the dust plasma [10]. In experiments, a Mach cone pattern in a dust crystalline layer created by the slow mode has been observed, corresponding to a shock speed of 23 mm/s [6].

The shock velocity of the slow mode of Eq. (17) may be rewritten as

$$U_- \simeq 2.3 \;\; T_{ev} a_{d\mu}^{-1} \vartheta^{-1/2} \left\{ \frac{F_\Phi(p)}{F_\Psi^2(p)} \frac{d[pF_\Phi(p)F_\Psi(p)]}{dp} + \frac{F_\Phi^2(p)}{F_\Psi(p)}\left(1 - \frac{1}{\gamma_d}\right)\right\}^{1/2} \text{(m/s)}. \tag{33}$$

Here, T_{ev} is the plasma temperature in electron volt, $a_{d\mu}$ is the dust average radius in μm, and ϑ is the dust specific weight. This equation gives a relationship between the shock velocity and the dust and plasma parameters. The shock velocity may be measured, for example, by the opening angle of a Mach cone created by a body moving through the dust plasma. This will lead to information on the dust and plasma parameters. This idea has been suggested to be used as a diagnostic method in space experiment such as *CASSINI to the Saturn* [4,9] and in dusty plasma experiments [5], to obtain information on dust plasma conditions where one cannot, or may with difficulty, obtain *in situ* observations.

(2) The electrostatic energy of the charged dust particles will affect the shock process and its properties. For the slow mode, as was shown in Sec. III, it determines the shock speed. For the fast mode, as was shown in Sec. IV, it has an effect on the energy conversion of the shocked flow.

(3) An equation, derived from the above set of macroscopic Eqs. (1)–(4) and (18)–(20), which corresponds to the hydrodynamic equation known as the *Rankine-Hugoniot* equation describing the energy conversion of the shocked flow, is now

$$\{I\} + \frac{1}{2}(P_1 + P_2)\{V\} + \frac{\alpha}{2}(P_{d1} + P_{d2})\{V_d\} + \alpha\{I_d\} = 0. \tag{34}$$

Here,

$$\alpha\{I_d\} = \alpha\frac{a_0^2}{4\pi}\frac{a_d}{M_d e^2}(\kappa T_1)^2\left[\left(\frac{T_2}{T_1}\right)^2 - 1\right]. \tag{35}$$

The second and third terms of Eq. (34) are negative as in normal shocks, the fourth one, which comes from the dust charge variations, is positive as shown by Eq. (35). This term describes how the flow of energy is now converted partly into electrostatic energy of the dust particles as they are charged up. Assuming $T_1 = 1$ ev, a radius of the dust particle $a_d = 1$ μm, and $a_0 = 2.50$ as in a hydrogen plasma, one obtains

$$\alpha\{I_d\} = 1.09\times 10^3 \frac{n_{d1}}{2n_{i1}} P_1 V_1 \left[\left(\frac{T_2}{T_1}\right)^2 - 1\right].$$

Compared with the other terms of Eq. (34), this has a considerable effect on the energy balance in the fast mode shock. This energy may, in principle, also be transferred into other forms of energy, for example, causing the electric fragmentation of the dust grain.

(4) In this paper only the charged particles are considered. For a partly ionized plasma, the collisions with neutrals may have an effect on the shock. In the Appendix, we compare the effect of collisions with neutrals with that of the electric field and find that these collisions may have a small effect on the shock over a wide range of the parameters.

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APPENDIX

For an incompletely ionized plasma, the effects of collisions with neutrals may affect the shock. It may change the shock model and it will also affect the shock structure when the mean free path of the neutrals are smaller than or comparable with the shock thickness.

Taking the collisions with the neutrals into consideration, the momentum equation of the plasma fluid becomes

$$D\frac{d\vec{v}}{dt}+\nabla\cdot\vec{P}-\rho\vec{E}+D\nu_{in}(\vec{U}-\vec{U_n})=0, \tag{A1}$$

where ν_{in} describes the ion-neutral coupling

$$\nu_{in}=2.41\pi\left(\frac{m_i m_n}{m_i+m_n}\right)^{1/2}\frac{D_n}{m_i m_n}e\alpha_p^{1/2}. \tag{A2}$$

Here, α_p is the polarizability of the neutral species, $\alpha_p=8.04\times10^{-25}$ cm^3 for H_2, D_n, and m_n is the density and the mass of the neutrals respectively [11]. Comparing the last and the third term on the left-hand side of Eq. (A1) and using Eqs. (16), (A2), and Table I, we obtain the ratio of the collision term to the electric field term for a hydrogen plasma. It is

$$\mathrm{Re}=0.908\times10^{-10}\times\frac{1-\eta}{\eta}\times\beta\times T_S\times\Delta u, \tag{A3}$$

where $\eta=n/2N$ is the ionization fraction of the plasma ($n/2$ is the density of ionized atoms and N the density of ionized plus neutral atoms), $\beta=m_iU^2/k_BT$ is the ratio of fluid energy to the thermal energy, characteristic time of shock $T_S=L/U$ (L the characteristic length of shock) in units of second, and $\Delta u=|U-U_n|/U$, the relative velocity difference of the neutrals to the plasma. Effects of collisions with neutrals is negligible if $\mathrm{Re}\ll1$. It could happen in a rather wide range of the parameters. In dust experiments, we generally have that $\eta\geq10^{-7}$, while the other parameters are of the order of one, so $\mathrm{Re}\ll1$. In planetary rings $\eta\sim1$ and Re will also be $\ll1$. However, in interstellar molecular clouds, we may have $\eta\sim10^{-8}$, while $T_s\gg1$ so that collisions with neutrals may have a large effect.

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